Computing Replenishment Cycle Policy Parameters for a Perishable Item*

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Abstract. In many industrial environments there is a significant class of problems for which the perishable nature of the inventory cannot be ignored in developing replenishment order plans. Food is the most salient example of a perishable inventory item. In this work, we consider the periodic-review, single-location, single-product production/inventory control problem under non-stationary stochastic demand and service level constraints. The product we consider can be held in stock for a limited amount of time after which it expires and it must be disposed of at a cost. In addition to wastage costs, our cost structure comprises fixed and unit variable ordering costs, and inventory holding costs. We propose an easy-to-implement replenishment cycle inventory control policy that yields at most $2N$ control parameters, where N is the number of periods in our planning horizon. We also show, on a simple numerical example, the improvement brought by this policy over two other simpler inventory control rules of common use.

Keywords: inventory management, perishability, uncertainty and risk, optimization

1 Introduction

According to Dawson (2004), the availability and the assortment of fresh products have a significant influence on consumer choice. This is one of the reasons why food supply chain management has an increasingly strategic importance in retail competition. However, as pointed out by Lütke Entrup (2005), in available Advanced Planning Systems shelf-life aspects of food are not adequately incorporated. This is mainly due to the fact that inventory control of perishable products is, in general, a challenging task. Every month we observe billions of dollars of food expiring on supermarket shelves and at the same time supermarkets lose revenues for food products that are not available on the shelf. Gruen et al. (2002) report that worldwide the average out-of-stock rate is 8.3%, in Europe it is 8.6%, and in the US 7.9%. This situation reflects the classic newsvendor trade-off between ordering too much and observing wastage, or ordering too little and therefore facing lost sales.

Inventory problems of perishable products have been discussed extensively in the literature. The first and most cited review work on this topic is by Nahmias (1982), who provides a review of the early literature on ordering policies for perishable inventories between early 1960s and 1982. A more recent work is by Karaesmen et al. (2008), who review the supply chain management literature of perishable products having fixed or random lifetimes. They classify the literature into periodic and continuous review inventory control and for each category they provide a detailed classification concerning specific model assumptions, e.g., replenishment policy (optimal control policy, order-up-to policy, or heuristic), excess demand (backlogged or lost), and lead-time (zero or strictly positive).

From the two survey works discussed above, we can observe the fact that in the inventory modelling literature, predominantly a total profit maximization or total cost minimization approach is pursued. However, the structure of the optimal replenishment policy is typically complex: the replenishment quantity depends on the individual age categories of current inventories and all outstanding orders. van Zyl (1964) investigates a periodic review problem of a product having a two period life-time, zero lead time, and a FIFO issuing policy with the objective to minimize expected costs consisting of holding cost, shortage cost, and cost of outdated inventories and shows the existence of an optimal order-up-to policy. Pierskalla and Roach (1975); Nahmias and Pierskalla (1973), and Fries (1975) extend van Zyl's work and derive op-

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timal ordering policies for a general lifetime of N periods. Unfortunately, these are no longer base-stock policies. Therefore, simplifying heuristic ordering policies have received much attention, e.g., Cohen (1976) analyzes a single critical number policy, whereas Brodheim et al. (1975) present a Markov-chain analysis of a constant order policy (COP). Pierskalla and Roach (1975) show that FIFO is the optimal issuing policy from the perspective of the retailer when the objective is to minimize total inventory holding costs. All the above works ignore fixed ordering costs. Relaxing such an assumption clearly may only complicate further the structure of the optimal policy. For this reason, according to Karaesmen et al. (2008), developing effective heuristic policies is of great practical importance in periodic-review inventory systems for perishable items in which a positive fixed ordering cost is considered. Nahmias (1978) points out that in practice it is far more realistic to allow the ordering cost to be composed of both fixed and proportional components. In the same work he establishes, computationally, that while not strictly optimal, (s,S) type policies perform very close to optimal in this case. Moreover, he also presents two methods of approximating the optimal (s,S) parameters.

Nahmias (1978) considers a problem in which the so-called "standard" cost structure is adopted, which is composed by fixed and variable ordering costs, wastage costs, holding and shortage costs. However, nowadays it is widely known and accepted that stock-out penalty cost approaches are difficult to implement and hardly used in practice, mainly due to the estimation of their value, e.g., how to quantify the loss of goodwill. In fresh food retail, this is particularly accentuated by cross selling arguments, i.e., the unavailability of fresh food products impacts the sales of other product categories and even mid- and long-term outlet choice. Therefore, a service level approach becomes essential for fresh food inventory management. However, according to the two surveys mentioned above, in the inventory control literature, this approach has been rarely analyzed so far.

One of the pioneering works in the application of a service level oriented approach in inventory control of perishable products is by Minner and Transchel (2010), who discusses a periodic-review, single-product service-level approach for perishable item inventory control with positive lead times, fixed shelf-life, different inventory issuing policies, and non-stationary demands under the lost sales assumption. Nevertheless, one of the simplifying assumptions adopted in this work is that the fixed ordering costs are negligible.

In this work, we analyze a periodic-review, single-location, single-product production/inventory control problem under non-stationary stochastic demand and service level constraints for perishable item. In contrast to Minner and Transchel (2010), who assume lost sales, we operate under a complete back-order assumption. Our cost structure considers fixed and variable ordering costs, wastage costs and holding costs. We propose an easy to implement "replenishment cycle", i.e. (R^n, S^n), policy (see Silver et al. (1998)), which yields at most $2N$ control parameters. The motivation for the (R,S) policy is given in Li et al. (2009). It is pointed out that in many logistics systems inventory replenishment by retailers consists of two distinct stages. In the first stage, retailers may make a delivery request in advance without making a firm commitment on order quantity. In the second stage, the retailers are allowed to make an adjustment to the quantity requested right before the shipping schedule is finalized. This approach allows retailers to postpone their quantity decision until better demand information is available, and, as a result, the mismatch between supply and demand may be reduced.

The work is structured as follows. In Section 2 we provide the problem definition and the modelling assumptions. In Section 3 a stochastic programming formulation of the problem is discussed. In Section 4 we introduce the deterministic equivalent model for computing the policy parameters. In Section 5 we provide a numerical example. In Section 6 we draw conclusions.

2 Problem definition

We consider the single-location, single-product production/inventory control problem under non-stationary stochastic demand and service level constraints. The product we consider can be held in stock for a limited amount of time after which it expires and it must be disposed of at a cost.

We consider a planning horizon of N periods and a demand d_t for each period $t \in \{1,\ldots,N\}$, which is a non-negative random variable with known probability density function $g_t(d_t)$. We assume that the demand occurs instantaneously at the beginning of each time period. The demand is non-stationary, that is it can vary from period to period and demands in different periods are assumed to be independent. Demands occurring when the system is out of stock are back-ordered and satisfied as soon as the next replenishment order arrives. The sellback of excess stock is not allowed. A fixed delivery cost a and a proportional unit cost u are incurred for each order. A replenishment order is assumed to arrive instantaneously at the beginning of each period, before the demand in that period occurs. For ease of exposition, we assume that there is no replenishment lead-time; however, the model can be easily extended to systems with positive replenishment lead-times. Each item that is delivered by the supplier arrives fresh and expires in exactly $M+1$ periods; therefore a product age may range from 0 to M. A linear holding cost h is incurred for each unit of product carried in stock from one period to the next. A linear wastage cost w is incurred, at the end of each period, for each unit of product that reached age M. Our aim is to find a replenishment plan that minimizes the expected total cost, which is composed of ordering costs, holding costs, and wastage costs over an N-period planning horizon, satisfying service level constraints. As a service level constraint we require that, with a probability of at least a given value α, at the end of each period the net inventory will be non-negative.

The actual sequence of actions is to some extend arbitrary. In what follows, we will assume that at the beginning of a period, the inventory on hand after all the demands from previous periods have been realized is known, for each product age that is available. Since we are assuming complete backlogging, this quantity may be negative. However, note that only fresh products can be backordered, since the supplier only delivers fresh products. On the basis of this information, an ordering decision is made for the current period and the respective order is immediately received. Then the period demand is observed and the stock is reduced according to a FIFO issuing policy. If, after the demand has been observed, there are still items of age M in stock, these are disposed at cost w per unit. Finally, holding cost is incurred on the remaining stock that is carried over to the next period.

3 Stochastic Programming Model

In this section, we extend the chance-constrained model originally proposed in Bookbinder and Tan (1988) in order to relax the assumption that items can be held in stock for a potentially unlimited amount of time. As pointed out in Tempelmeier (2008), since period demands are random, the net inventory may become negative. However, the number of stock-outs is restricted by the service level constraints enforced. While computing holding costs, we will assume, as in Tempelmeier (2008), that the service level is set large enough to ensure that the net inventory will be a good approximation of the inventory on hand.

The modified chance-constrained model is as follows,

$$\min \int_{d_1} \cdots \int_{d_N} \sum_{t=1}^{N} \left(a\delta_t + h\sum_{i=0}^{M-1} I_t^i + uQ + wI_t^M \right) g(d_1)\ldots g(d_N)\mathrm{d}d_1 \ldots \mathrm{d}d_N \tag{1}$$

subject to

$$\delta_t = \begin{cases} 1 & Q_t > 0 \\ 0 & otherwise \end{cases} \qquad t = 1,\ldots,N \tag{2}$$

$$\sum_{i=0}^{M} I_t^i + d_t - \sum_{i=0}^{M-1} I_{t-1}^i = Q_t \qquad t = 1,\ldots,N \tag{3}$$

$$I_t^i = \max\left\{ I_{t-1}^{i-1} - \max\left\{ d_t - \sum_{j=i}^{M-2} I_{t-1}^j, 0 \right\}, 0 \right\} \qquad t = 1,\ldots,N \quad i = 1,\ldots,M \tag{4}$$

$$\Pr\left\{ \sum_{i=0}^{M} I_t^i \geq 0 \right\} \geq \alpha \qquad t = 1,\ldots,N \tag{5}$$

$$I_0^i = 0 \qquad i = 0,\ldots,M-1 \tag{6}$$

$$I_t^i \geq 0 \qquad t = 1,\ldots,N \quad i = 1,\ldots,M \tag{7a}$$

$$I_t^0 \in R \qquad t = 1,\ldots,N \tag{7b}$$

$$\delta_t \in \{0,1\} \qquad t = 1,\ldots,N \tag{8}$$

$$Q_t \geq 0 \qquad t = 1,\ldots,N \tag{9}$$

In contrast to Bookbinder and Tan's model, in our model we track inventory of different ages via dedicated decision variables and issuing policy constraints. More specifically, (6) $I_0^i = 0$ denotes the initial inventory of age $i \in \{0,\ldots,M-1\}$, without loss of generality here assumed to be zero; (7a) I_t^i

denotes the inventory level of age $i \in \{1,\ldots,M\}$ at the end of period $t \in \{1,\ldots,N\}$, a random variable that takes non-negative values, since non-fresh products cannot be back-ordered; (7b) I_t^0 denotes the inventory level of age 0 at the end of period $t \in \{1,\ldots,N\}$, a random variable that takes real values, since fresh products can be back-ordered; (8) δ_t is a binary decision variables that is set to 1 if and only if an inventory review is scheduled in period t; (9) $Q_t \geq 0$ denotes the order quantity of fresh items in period t. The objective function (1) minimizes the expected total cost. Eq. (2) states that a review is scheduled, and the respective fixed cost is incurred in the objective function, whenever the order quantity is positive at a given period t. Eq. (3) relates the order quantity in period t, Q_t, to the amount of inventory that is carried over from period $t-1$ to period t. Eq. (4) implements the FIFO issuing policy and the respective rule for the „consumption" and the „aging" of existing stocks. Finally, Eq. (5) enforces the required service level in each period.

Different inventory control policies can be implemented for the above model. Bookbinder and Tan (1988) define the „static-dynamic uncertainty" strategy. In the inventory control literature, this strategy is better known as the non-stationary (*R*, *S*) policy. This policy consists of a series of review times (*R* denoting the number of periods between two consecutive replenishments), and associated order-up-to-levels *S*, all fixed at the beginning of the planning horizon (i.e., the static aspect of the strategy). However, the actual order quantities are determined only after observing the realised demand (i.e., the dynamic aspect of the strategy). It should be noted that, in our problem, only fresh items can be ordered from the supplier. If we define the inventory level at the beginning of period t, before any order is placed, as the total amount of stock carried over from the previous period $t-1$, that is $\sum_{i=0}^{M-1} I_{t-1}^i$, this easy to implement strategy, which yields at most $2N$ control parameters, can be immediately translated to the case in which a perishable product and a FIFO issuing policy are considered. Following the modelling strategy discussed in Tarim and Kingsman (2004) and Tarim and Smith (2008), in the next section we propose a deterministic equivalent Constraint Programming model that solves the above problem under the Bookbinder–Tan "static dynamic uncertainty" strategy (i.e., non-stationary (*R*, *S*) policy) with $2N$ control parameters.

4 Deterministic equivalent Constraint Programming model

Firstly, we recall that a (finite domain) constraint satisfaction problem (CSP) can be expressed in the following form. Given a set of variables, together with a finite set of possible values that can be assigned to each variable, and a list of constraints, find values of the variables that satisfy every constraint. Constraint Programming is an approach to solving decision and optimisation problems, historically arising in the field of Artificial Intelligence. Constraint programming (CP) allows variables, their possible values, and constraints on those variables, to be expressed and provides search algorithms for solving the resulting CSP. The constraint satisfaction framework can be extended to optimisation, for instance by expressing the objective as a dynamic constraint that becomes tighter as successive solutions are found. A comprehensive and up-to-date survey of the state of knowledge regarding CP is given by Apt (2003).

In Tarim and Kingsman (2004) the authors describe in detail how to implement a non-stationary (R, S) policy within a model similar to the one presented in Section 3, but that does not consider a perishable item. The steps presented in their work can be easily adapted to the case in which we must handle a perishable product. The resulting model is shown below.

$$\min \sum_{t=1}^{N} \left(a\delta_t + h\sum_{i=0}^{M-1} \tilde{I}_t^i + w\tilde{I}_t^M \right) + u\sum_{i=0}^{M} \tilde{I}_N^i \tag{10}$$

subject to

$$\left(\sum_{i=0}^{M} \tilde{I}_t^i + \tilde{d}_t - \sum_{i=0}^{M-1} \tilde{I}_{t-1}^i \right) \Rightarrow \delta_t = 0 \qquad t = 1,\ldots,N \tag{11}$$

$$\sum_{i=0}^{M} \tilde{I}_t^i + \tilde{d}_t - \sum_{i=0}^{M-1} \tilde{I}_{t-1}^i \geq 0 \qquad t = 1,\ldots,N \tag{12}$$

$$\tilde{I}_t^i = \max\left\{ \tilde{I}_{t-1}^{i-1} - \max\left\{ \tilde{d}_t - \sum_{j=i}^{M-2} \tilde{I}_{t-1}^j, 0 \right\}, 0 \right\} \qquad t = 1,\ldots,N \quad i = 1,\ldots,M \tag{13}$$

$$\sum_{j=i}^{M} \tilde{I}_{t-1}^{j} \geq \text{bufferMatrix}\left[\max_{j\in\{1,\ldots,t\}} j\delta_j, t\right] \qquad t = 1,\ldots,N \tag{14}$$

$$\tilde{I}_0^i = 0 \qquad i = 0,\ldots,M \tag{15}$$

$$\tilde{I}_t^i \geq 0 \qquad t = 1,\ldots,N \quad i = 0,\ldots,M \tag{16}$$

$$\delta_t \in \{0,1\} \qquad t = 1,\ldots,N \tag{17}$$

where

$$\text{bufferMatrix}[i,j] = G_{d_i+\ldots+d_j}^{-1}(\alpha) - \sum_{k=i}^{j} \tilde{d}_k \tag{18}$$

and $G_{d_i+\ldots+d_j}(.)$ is the cumulative distribution function of $d_i + \ldots + d_j$. It is assumed that $G_{d_i+\ldots+d_j}(.)$ is strictly increasing, therefore $G_{d_i+\ldots+d_j}^{-1}(.)$ is uniquely defined. $\tilde{I}_t^i$ is the expected inventory level of age $i \in \{0,\ldots,M\}$ at the end of period $t \in \{1,\ldots,N\}$.

The above model neatly resembles the CP model in Tarim and Smith (2008) and it is obtained by taking expectations on decision variables and by replacing Eq. (5) with the respective deterministic equivalent expression, Eq. (14), based on expected buffer stock levels pre-computed and stored in the bufferMatrix lookup table, i.e. Eq. (18). It should be noted that in Tarim and Kingsman (2004) in order to translate Eq. (5) into Eq. (14) the following assumption has been made. At a given review, if the actual stock exceeds the order-up-to-level, this excess stock is carried forward and it is not returned to the supply source. However, such occurrences are regarded as rare events and accordingly the cost of carrying this excess stock and its effect on the service levels of subsequent periods are ignored. The times of the stock reviews are given by the values of t such that $\delta_t = 1$. The associated order-up-to-levels, for each t, are given as $S_t = \tilde{d}_t + \sum_{i=0}^{M} \tilde{I}_t^i$. The above model can be easily extended to take a positive lead-time, l, into account by replacing (18) with $\text{bufferMatrix}[i,j] = G_{d_i+\ldots+d_j+d_{j+1}+\ldots+d_{j+l-1}}^{-1}(\alpha) - \sum_{k=i}^{j+l-1} \tilde{d}_k$. In what follows we will provide a numerical example solved by using the CP model above.

5 A numerical example

A single problem over an 8-period planning horizon is considered and the expected values for period demand, $\tilde{d}_t$, are $\{100,125,25,40,30,80,110,50\}$. The product we consider has a lifetime of 4 periods, therefore $M = 3$. We assume an initial null inventory level and a normally distributed demand for every period with a coefficient of variation $\sigma_t / \tilde{d}_t = 0.2$ for each $t \in \{1,\ldots,N\}$, where σ_t denotes the standard deviation of the demand in period t. We consider a fixed ordering cost value $a = 100$, a proportional unit cost $u = 5$, a proportional wastage cost $w = 2$ and a holding cost $h = 1$ per unit per period. The non-stock-out probability in each period is $\alpha = 0.95$. The policy parameters computed by using the model in Section 4 are shown in Table 1.

Table 1. Policy parameters

Period	1	2	3	4	5	6	7	8
$\tilde{d}_t$	100	125	25	40	30	80	110	50
S_t	133	192	-	86	-	106	146	66
$\tilde{I}_t^0$	33	67	0	44	2	26	36	16
$\tilde{I}_t^1$	0	0	42	0	14	0	0	0
$\tilde{I}_t^2$	0	0	0	2	0	0	0	0
$\tilde{I}_t^3$	0	0	0	0	0	0	0	0

The policy schedules 6 replenishment cycles whose lengths R_i are $\{1,2,2,1,1,1\}$ and the respective order-up-to-levels S_i are $\{133,192,86,106,146,66\}$, its expected total cost is 962. We compared the cost of our strategy with two other policies of common use in inventory control. As a result, we observed that this policy cost is roughly half that of a simple (R,Q) policy, for which the optimal values $R=4$ and $Q=344$, as well as the expected total cost 1947, have been obtained by exhaustive enumeration and simulation. This policy orders a constant amount Q of fresh items every R periods. Furthermore, the above policy cost is also roughly half than the cost of an (R,S) policy for which the optimal values $R=4$ and $S=340$, as well as the expected total cost 1888, have been obtained by exhaustive enumeration and simulation. This policy reviews the inventory every R periods, and it simply orders the amount of fresh items that is missing in order to raise the total inventory on hand, which is composed by all the non-expired product age that are on-hand at the beginning of a review period, up to level S.

6 Conclusion

In this work we addressed the periodic-review, single-location, single-product production/inventory control problem under non-stationary stochastic demand and service level constraints. The product we consider can be held in stock for a limited amount of time after which it expires and it must be disposed at a cost. In addition to wastage costs, our cost structure comprises fixed and unit variable ordering costs, and inventory holding costs. We proposed an easy-to-implement replenishment cycle inventory control policy that yields at most $2N$ control parameters, where N is the number of periods in our planning horizon. We also demonstrated the effectiveness of this policy, by means of a simple numerical example, over two other simpler inventory control rules of common use.

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